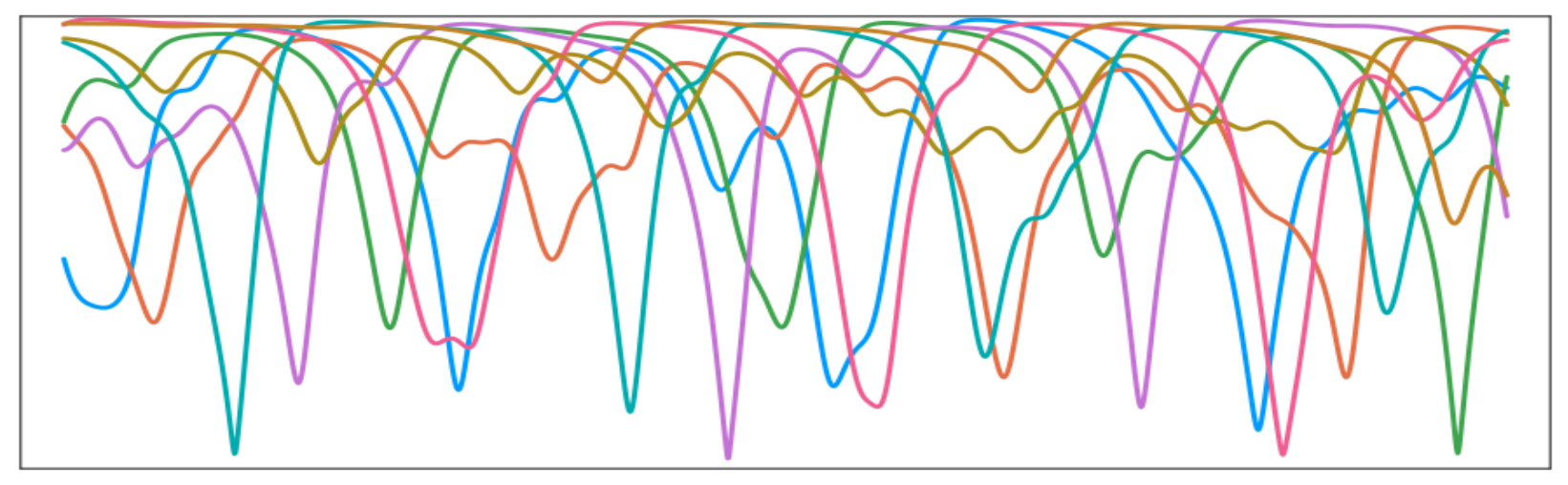
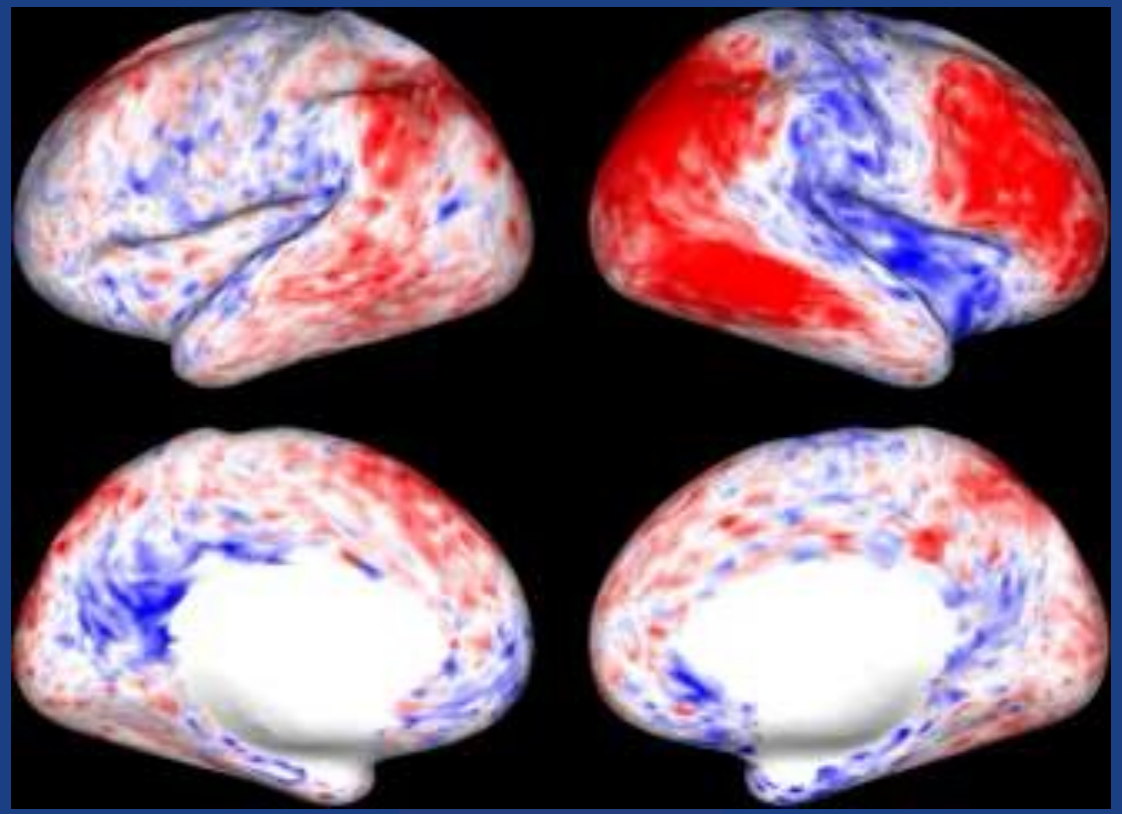
Emergence of metastability in frustrated oscillatory networks:
the key role of hierarchical modularity

Enrico Caprioglio & Luc Berthouze
Department of Informatics, University of Sussex



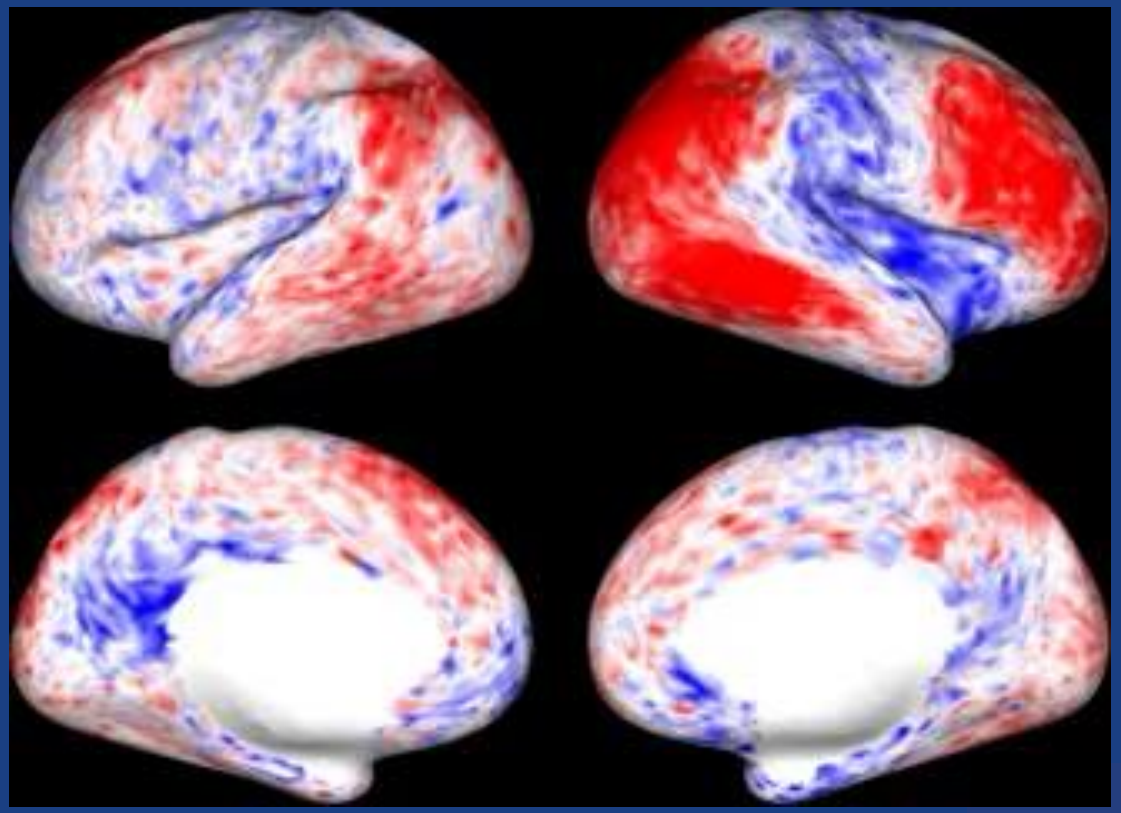
BRAINS vs ORCHESTRA



2x speed
Ref. [1]

[1]: Bhushan et al (2016). "Temporal Non-Local Means Filtering Reveals Real-Time Whole-Brain Cortical Interactions in Resting fMRI". PLOS ONE. DOI:10.1371/journal.pone.0158504

BRAINS vs ORCHESTRA



2x speed
Ref. [1]

https://www.clevelandorchestra.com/

[1]: Bhushan et al (2016). "Temporal Non-Local Means Filtering Reveals Real-Time Whole-Brain Cortical Interactions in Resting fMRI". PLOS ONE. DOI:10.1371/journal.pone.0158504

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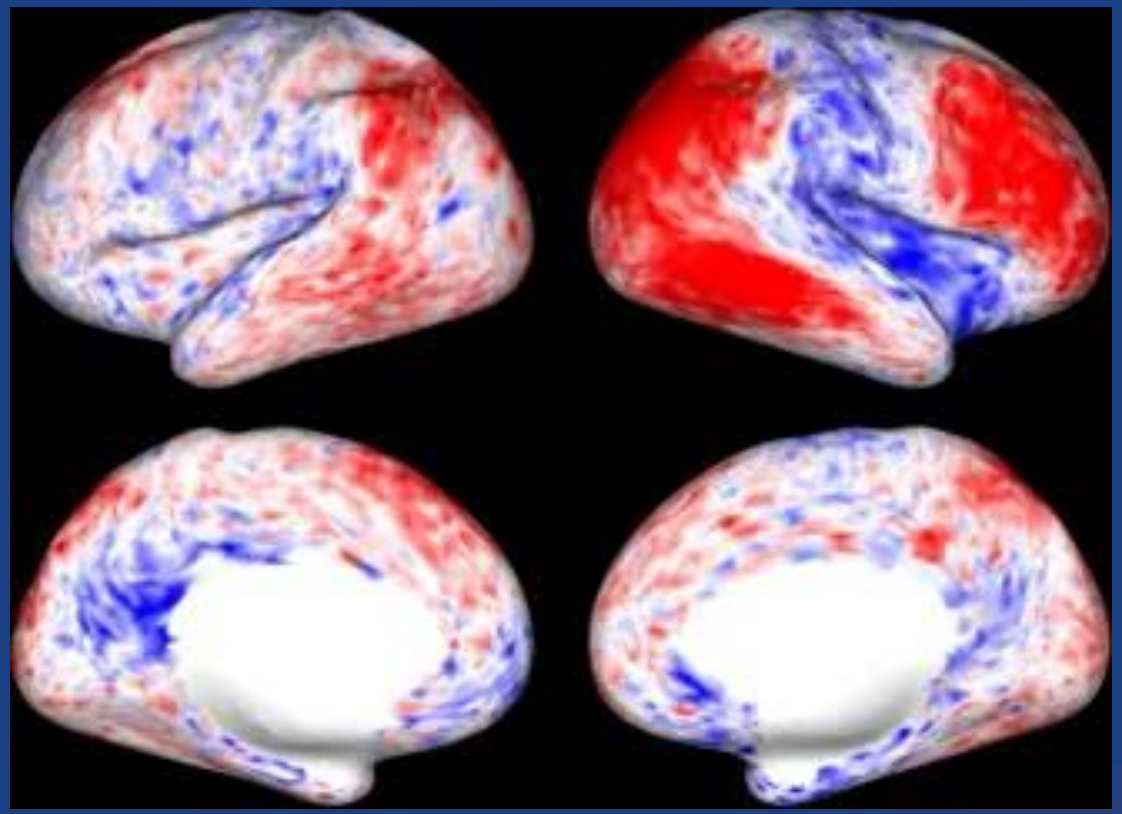

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*Oscillatory-based

METASTABILITY

a dynamical system that exhibits “unstable attractions”, in which segregative and integrative tendencies coexist [2]

[2] F. Hancock et al (2023) “Metastability Demystified — the Foundational Past, the Pragmatic Present, and the Potential Future.” Preprint MDPI AG

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WHOLE-BRAIN MODELS*

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METASTABILITY

a dynamical system that exhibits “unstable attractions”, in which segregative and integrative tendencies coexist [2]

WHOLE-BRAIN MODELS*

- is it actually metastable?
- many symmetry-breaking parameters
- is metastability critical?

[2] F. Hancock et al (2023) “Metastability Demystified — the Foundational Past, the Pragmatic Present, and the Potential Future.” Preprint MDPI AG

HYPOTHESIS

The (brain's) **hierarchically modular** mesoscale structure alone can give rise to **metastable dynamics** and robust chimera states in the presence of **phase frustration**

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HYPOTHESIS

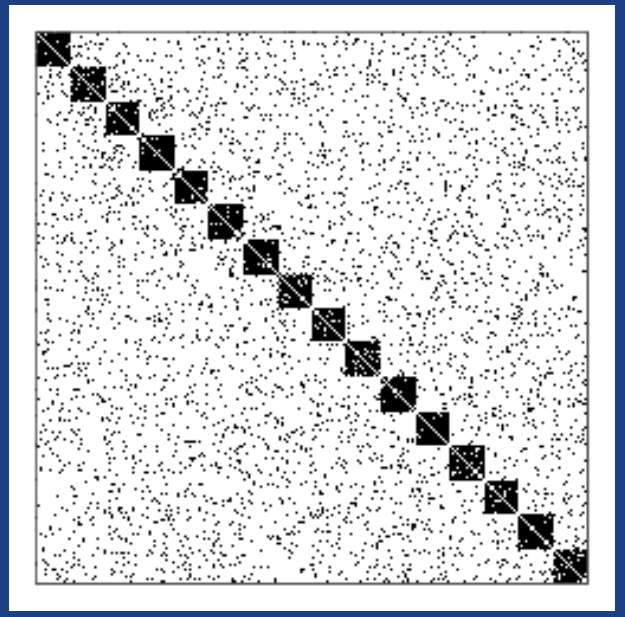
The (brain's) **hierarchically modular** mesoscale structure alone can give rise to **metastable dynamics** and robust chimera states in the presence of **phase frustration**

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \omega - K\sum_{j}^{N} A_{ij}\sin\left(\theta_j - \theta_i - \alpha\right)$$

Phase-frustrated **identical** oscillators

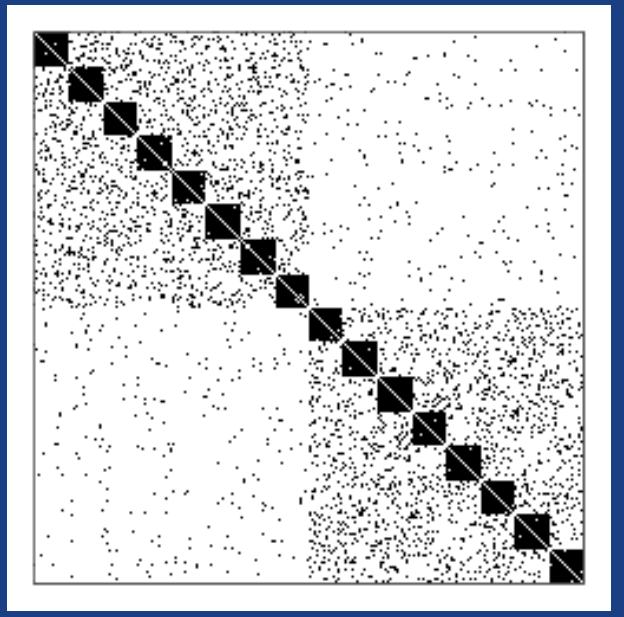
Crucially, in the absence of any other symmetry-breaking parameters

METHODS 1



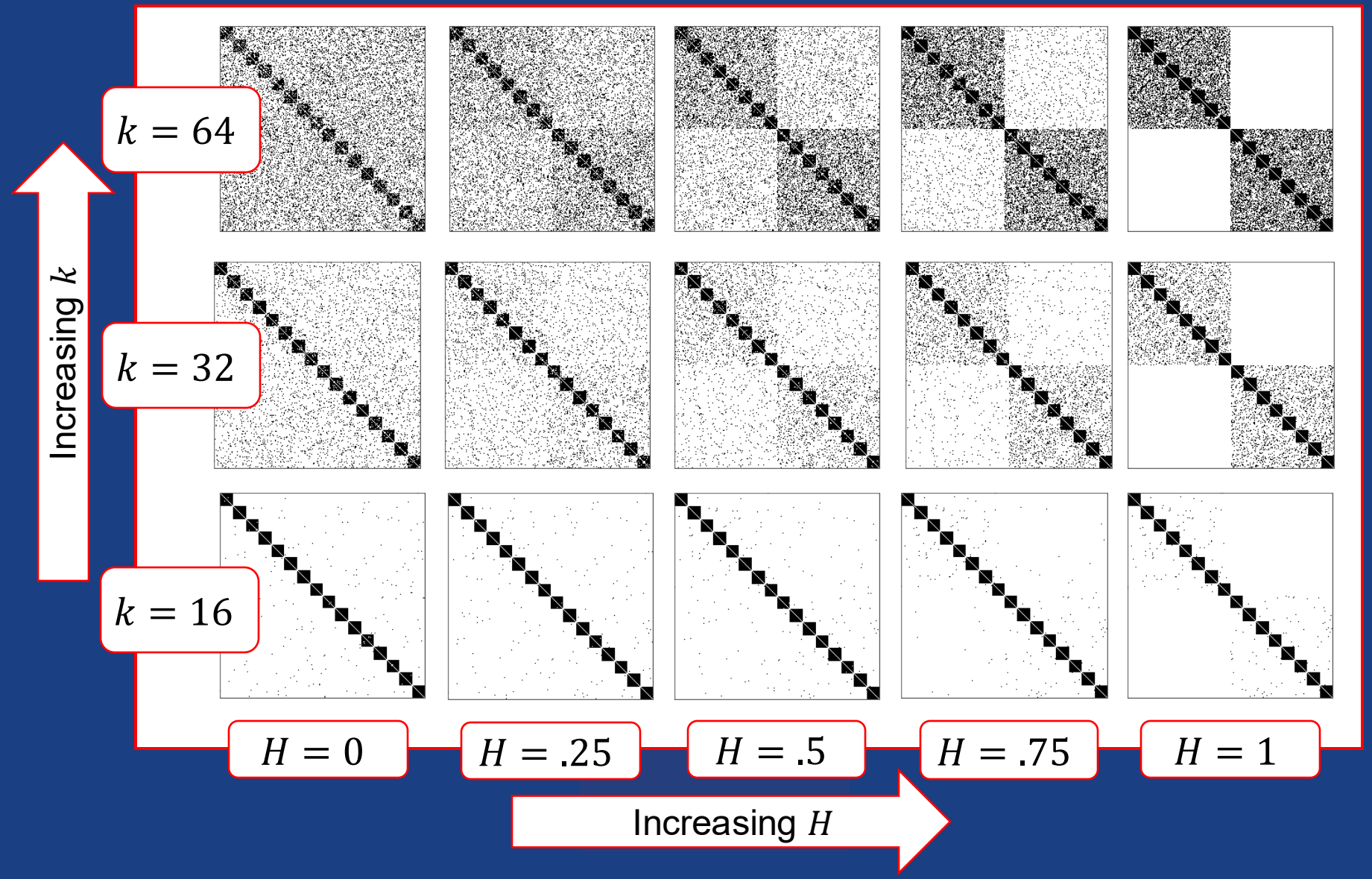
Community structured
(Stochastic Block Model)

vary H



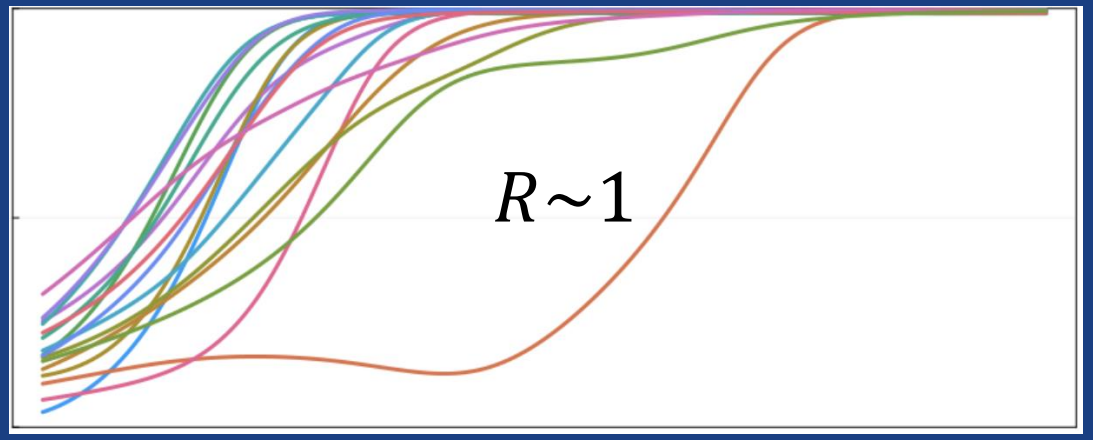
Hierarchically
modular network
(Nested Stochastic
Block Model)

METHODS 1

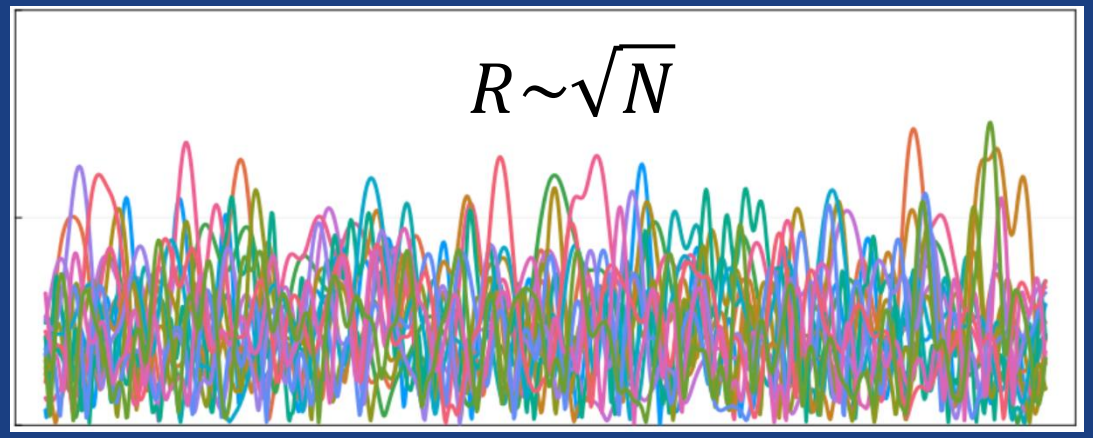


METHODS 2

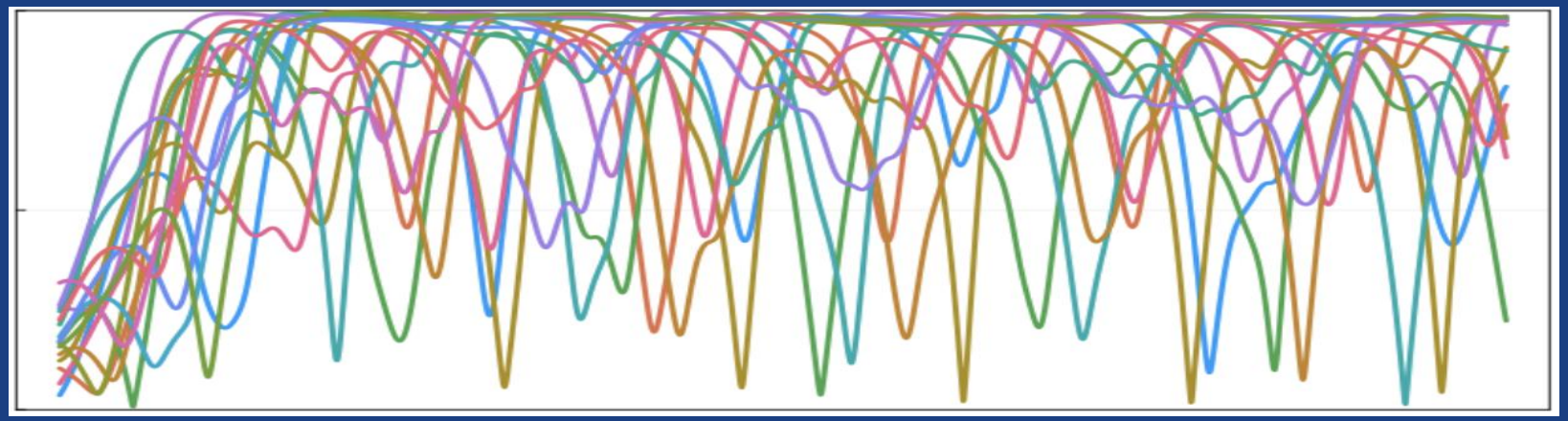
ORDER



DISORDER



METASTABILITY



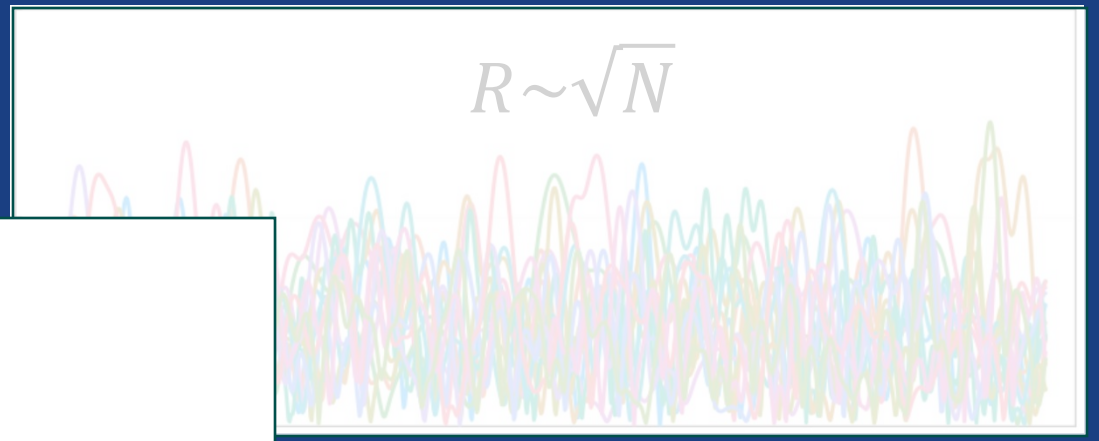
$$\sigma(R) > 0$$

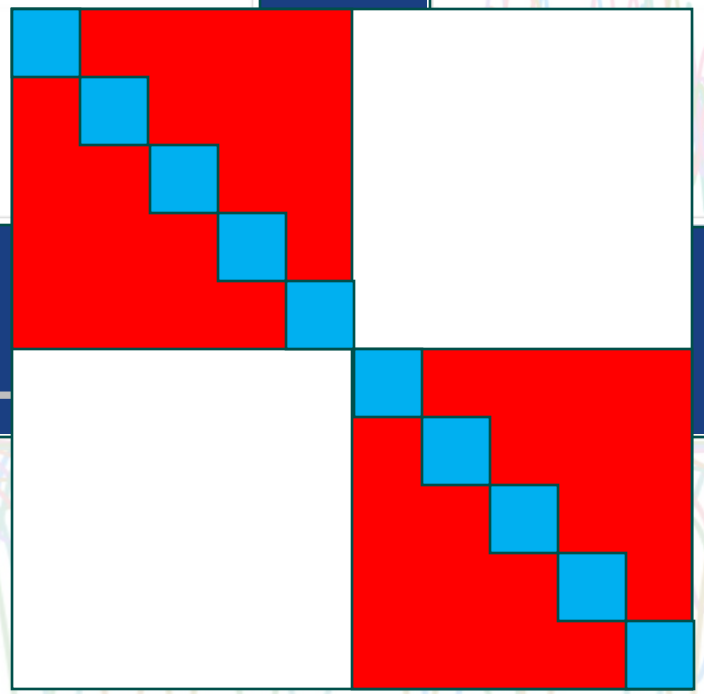
METHODS 2

[3] Arenas, A., Díaz-Guilera, A., & Pérez-Vicente, C. J. (2006). Synchronization Reveals Topological Scales in Complex Networks. In Physical Review Letters

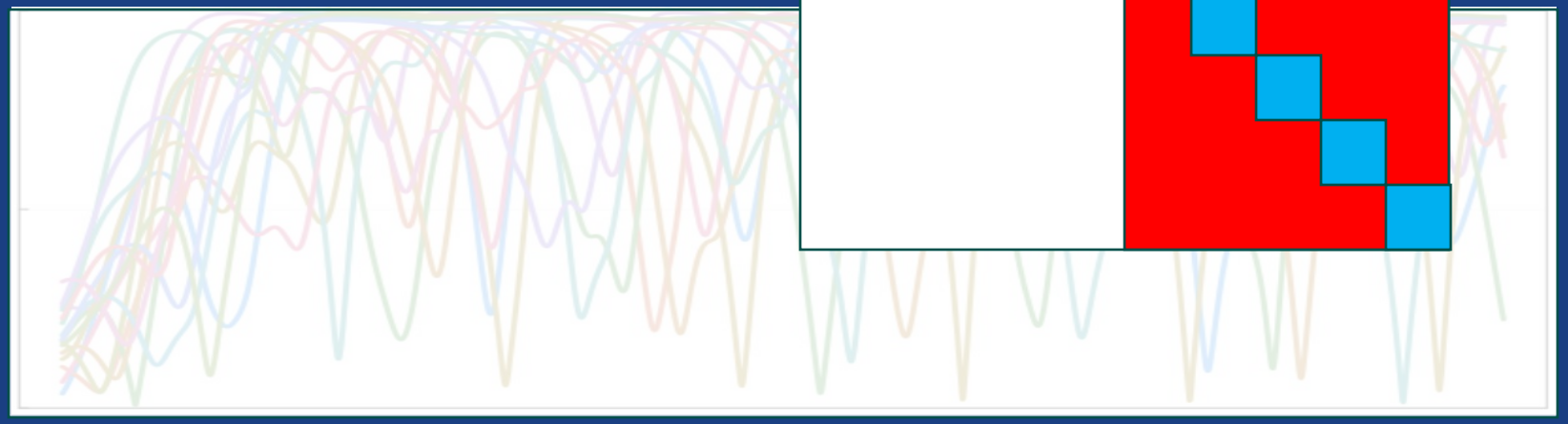
ORDER

DISORDER





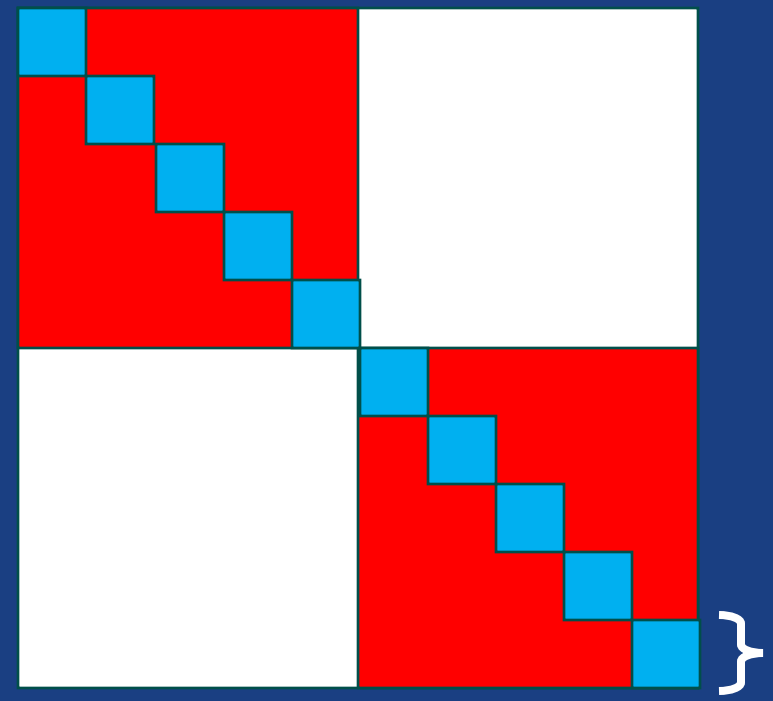
METASTABIL



$\sigma(R) > 0$

METHODS 2

ρ_i POPULATIONS



μ_i MODULES

Local Order Parameter

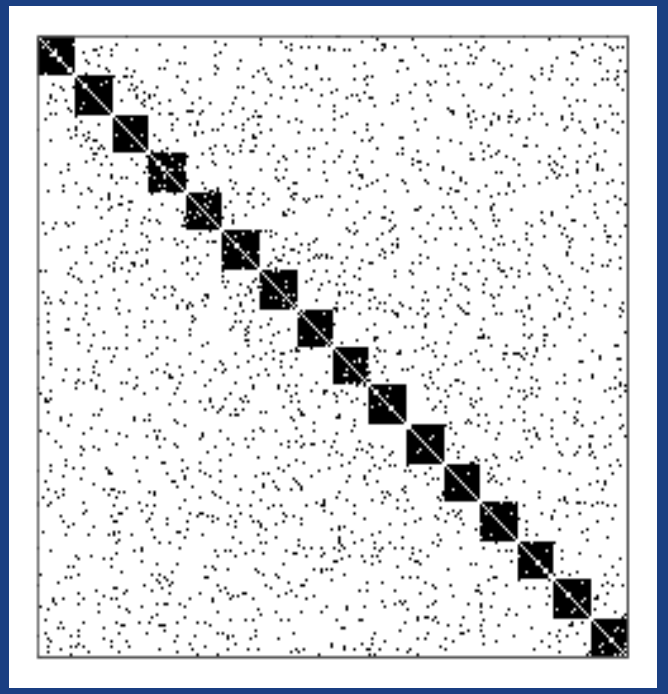
$$R_{\rho_i} = Re[\frac{1}{|\rho_i|}\sum_{j\in\rho_i}^{N}\exp(i\theta_j)]$$

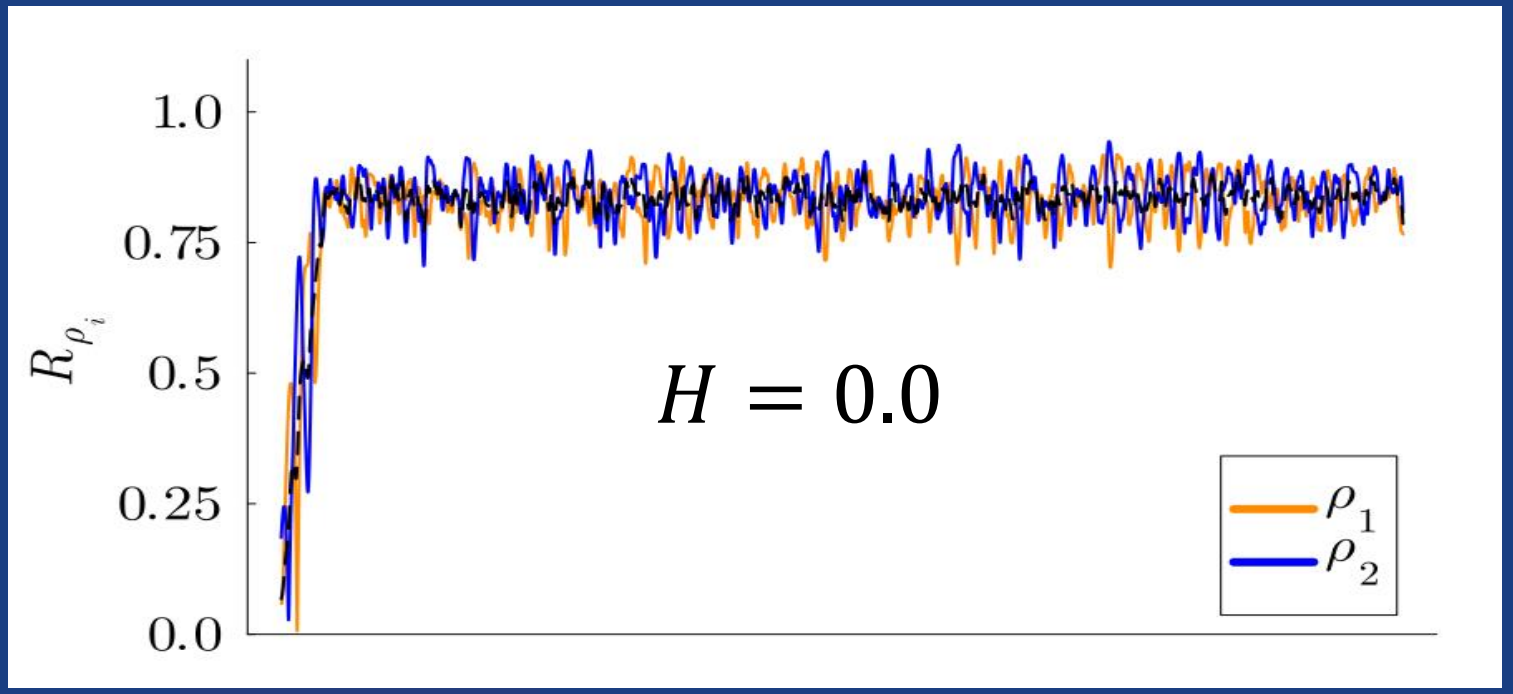
$$R_{\mu_i} = Re[\frac{1}{|\mu_i|}\sum_{j\in\mu_i}^{N}\exp(i\theta_j)]$$

METASTABILITY:

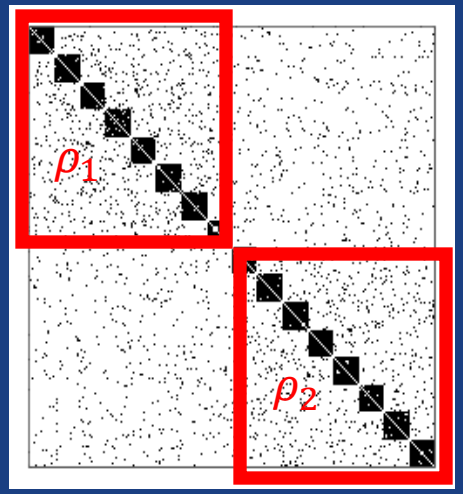
$$\sigma^2{}_{MET}(R_{\rho i}) = \langle\sigma(R_{\rho i})\rangle$$

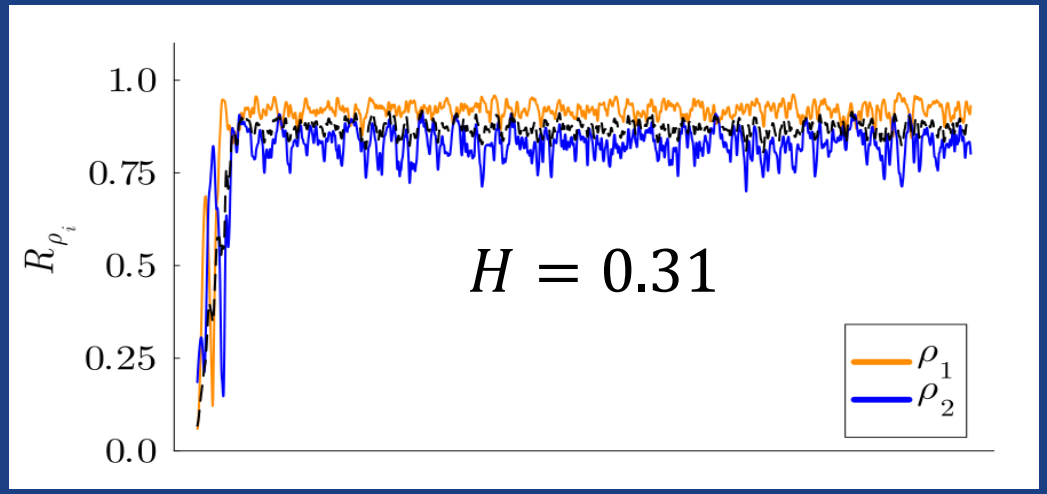
$$\sigma^1{}_{MET}(R_{\mu_i}) = \langle\sigma(R_{\mu_i})\rangle$$

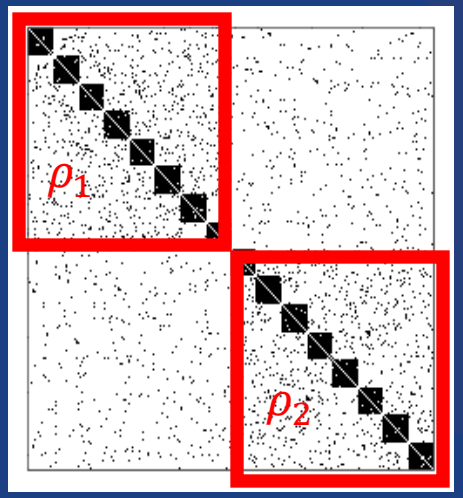


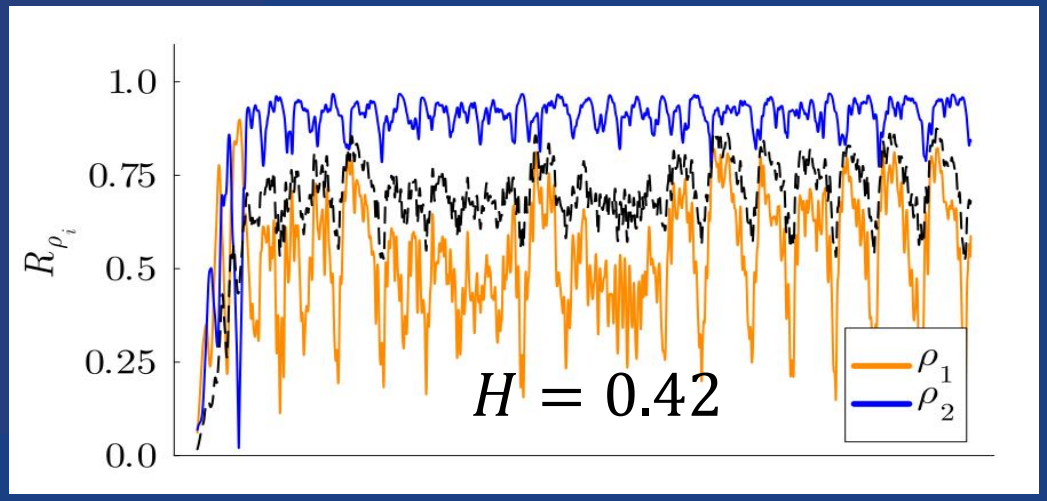

1.0
0.75
0.5
0.25
0.0
R_{ρ_i}
$H = 0.0$
ρ_1
ρ_2

STABLE AND BREATHING CHIMERAS EMERGE AT THE POPULATIONS' LAYER R_{ρ_1}, R_{ρ_2}

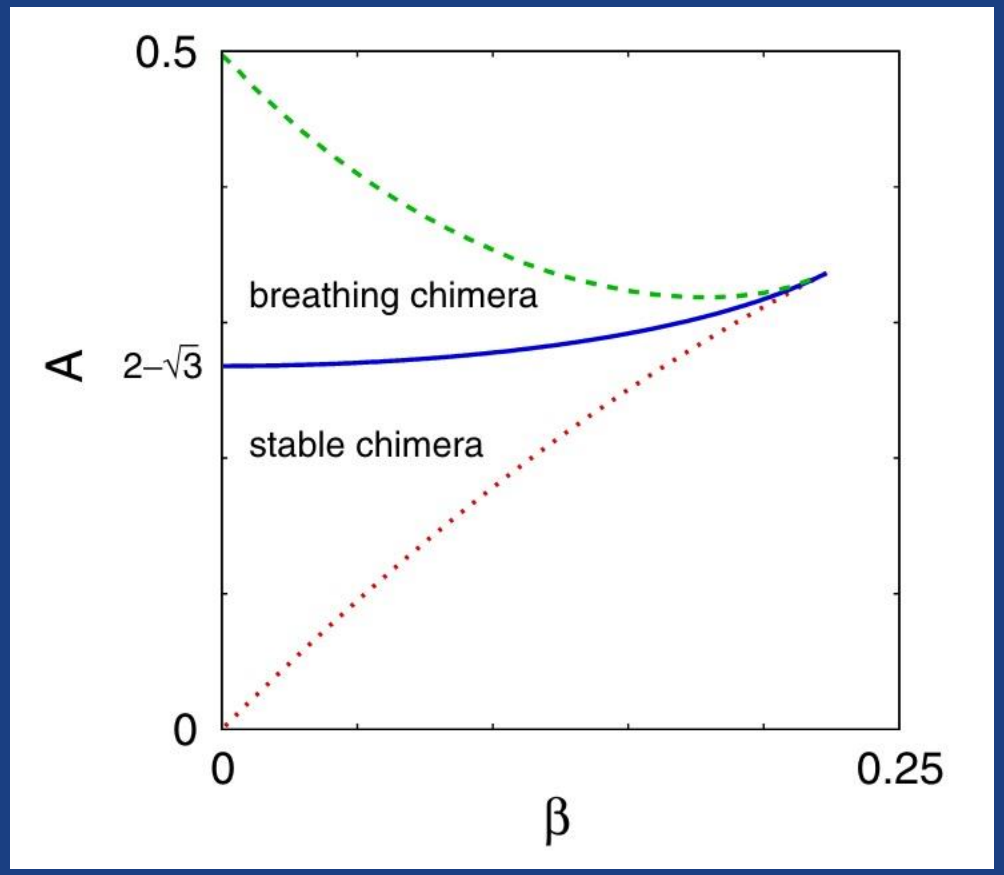


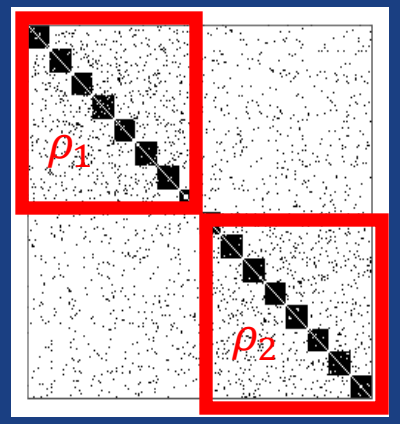


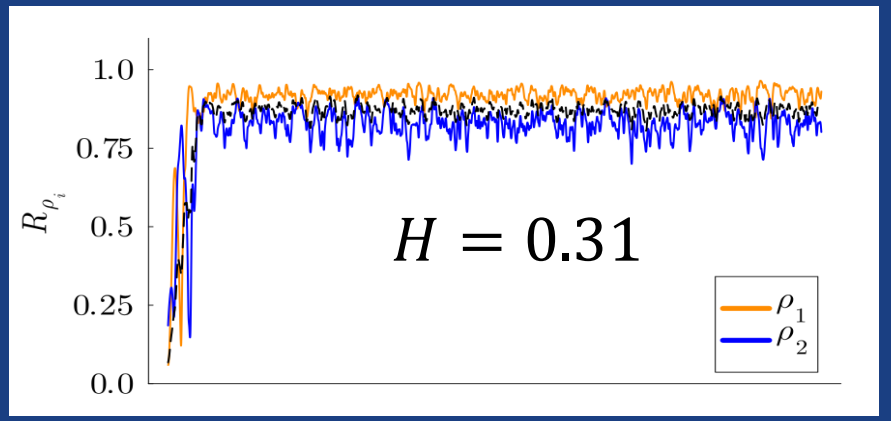


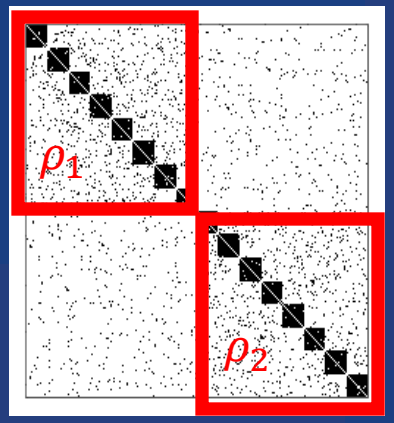


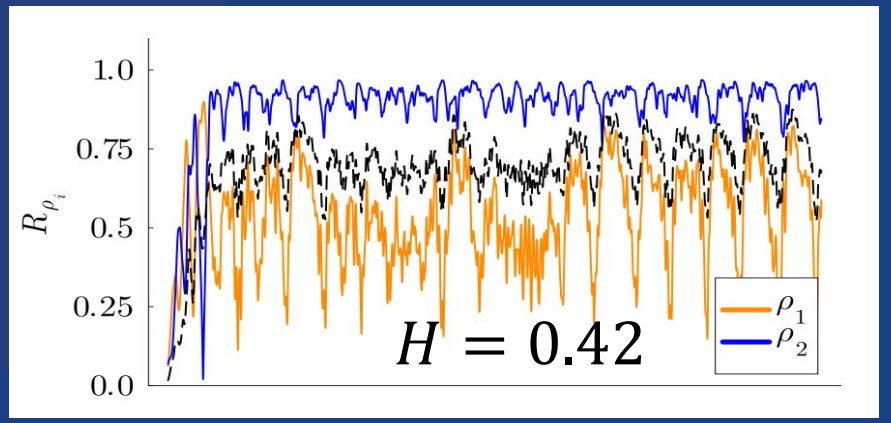
STABLE AND BREATHING CHIMERAS EMERGE AT THE POPULATIONS' LAYER R_{ρ_1}, R_{ρ_2}







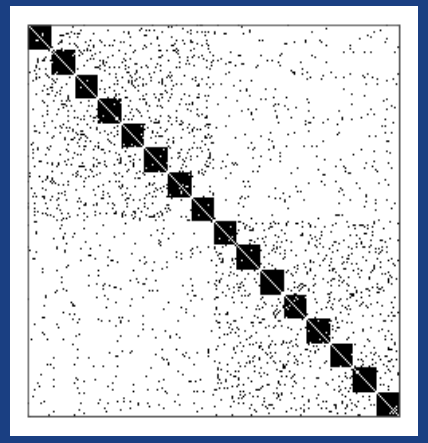


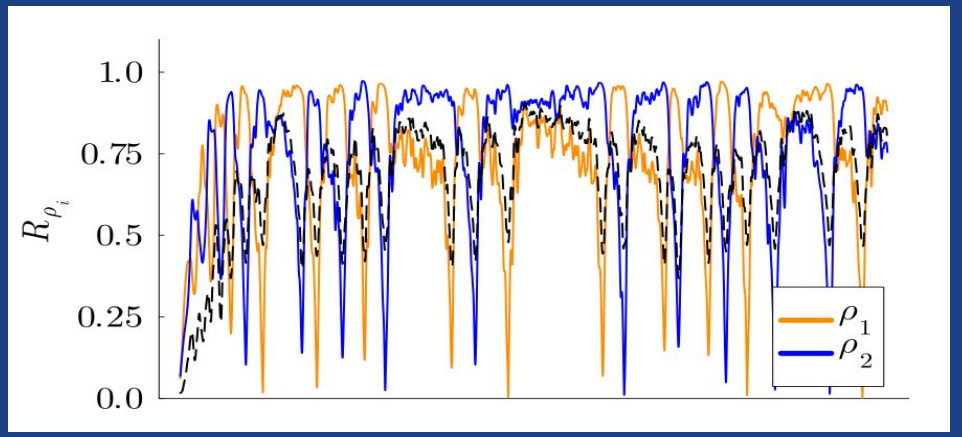


[4] D. M. Abrams, R. Mirollo, S. H. Strogatz, and D. A. Wiley, "Solvable Model for Chimera States of Coupled Oscillators," PRL

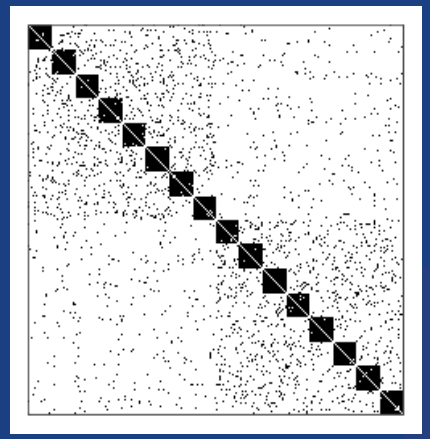
METASTABLE AND ALTERNATING CHIMERA STATES EMERGE AT THE POPULATION LEVEL

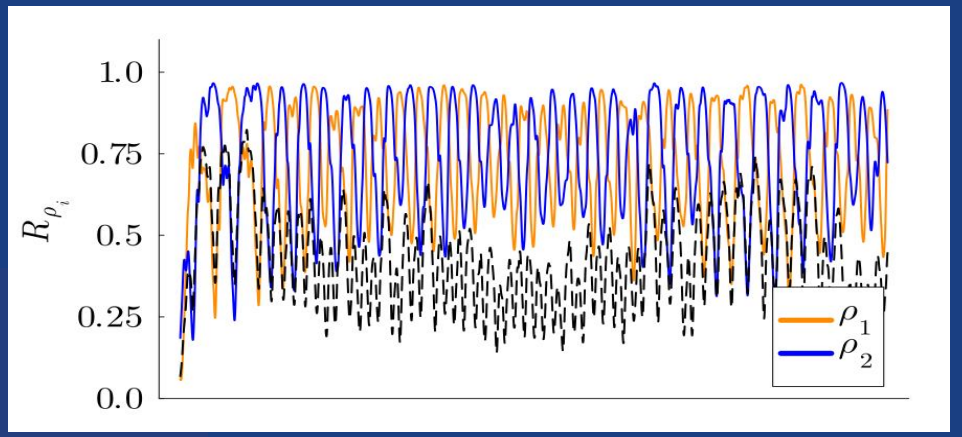
- $H = 0.5$





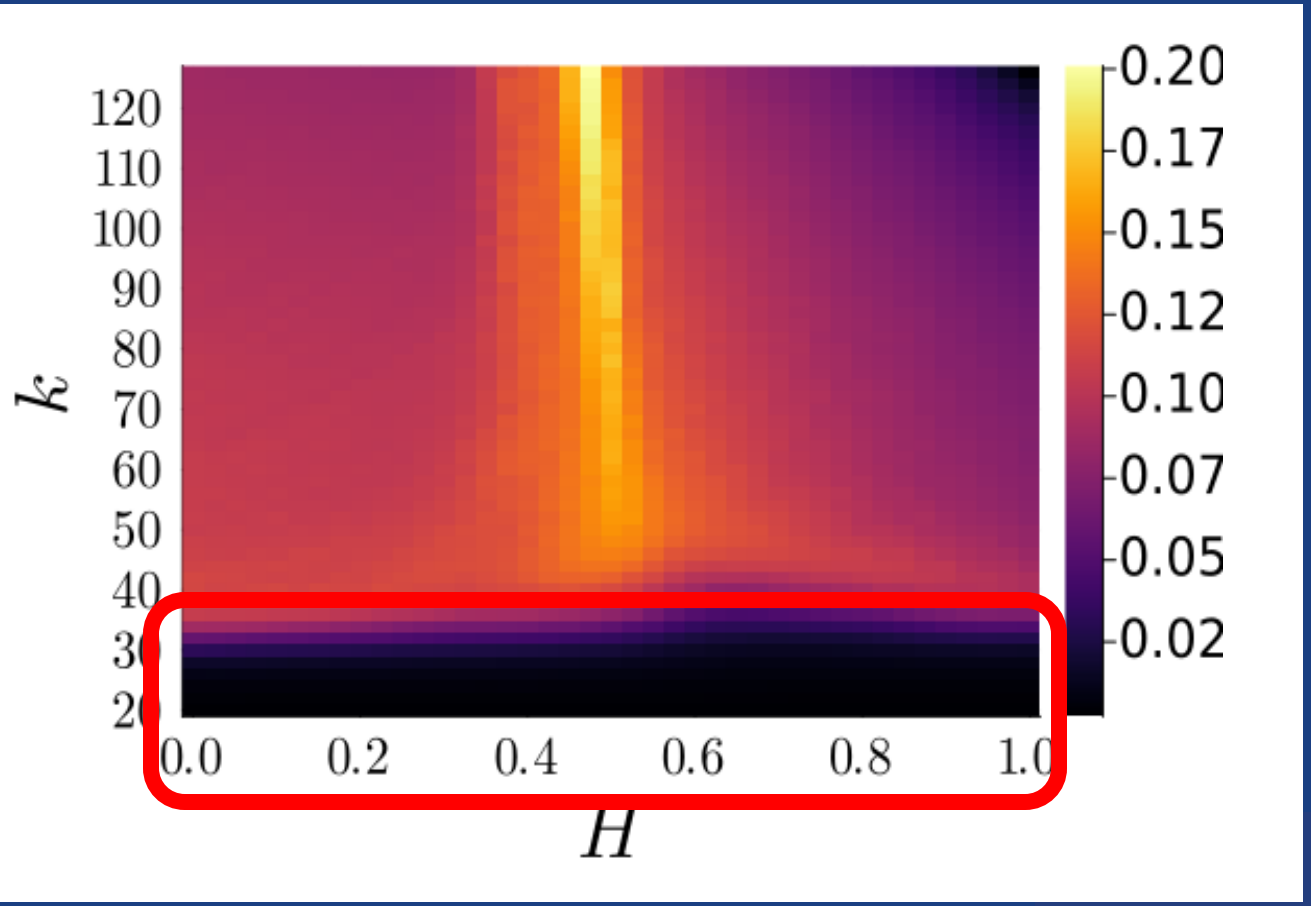
- $H = 0.55$

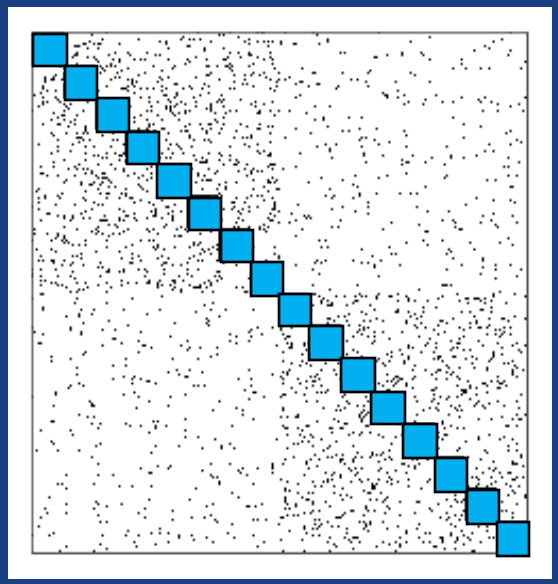




METASTABILITY: $\boldsymbol{\sigma_{MET}(R_{\rho i}) = \langle \sigma(R_{\rho i}) \rangle}$

METASTABLE DYNAMICS EMERGES AT THE MODULES' LAYER





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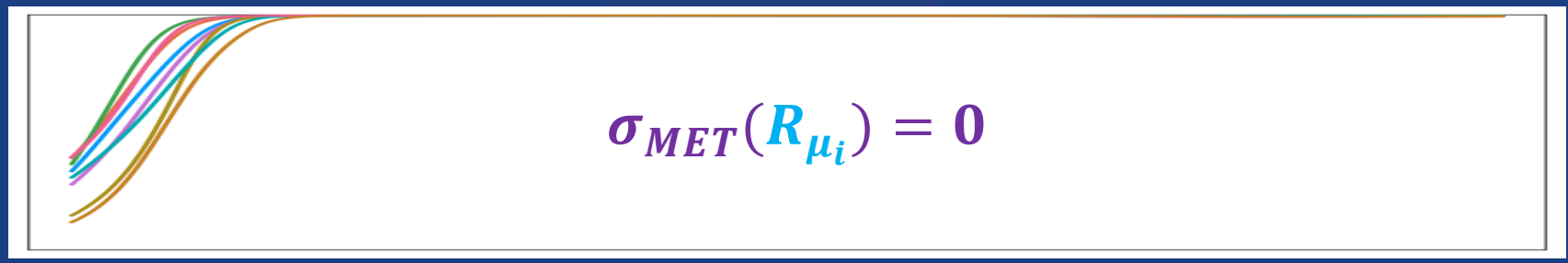
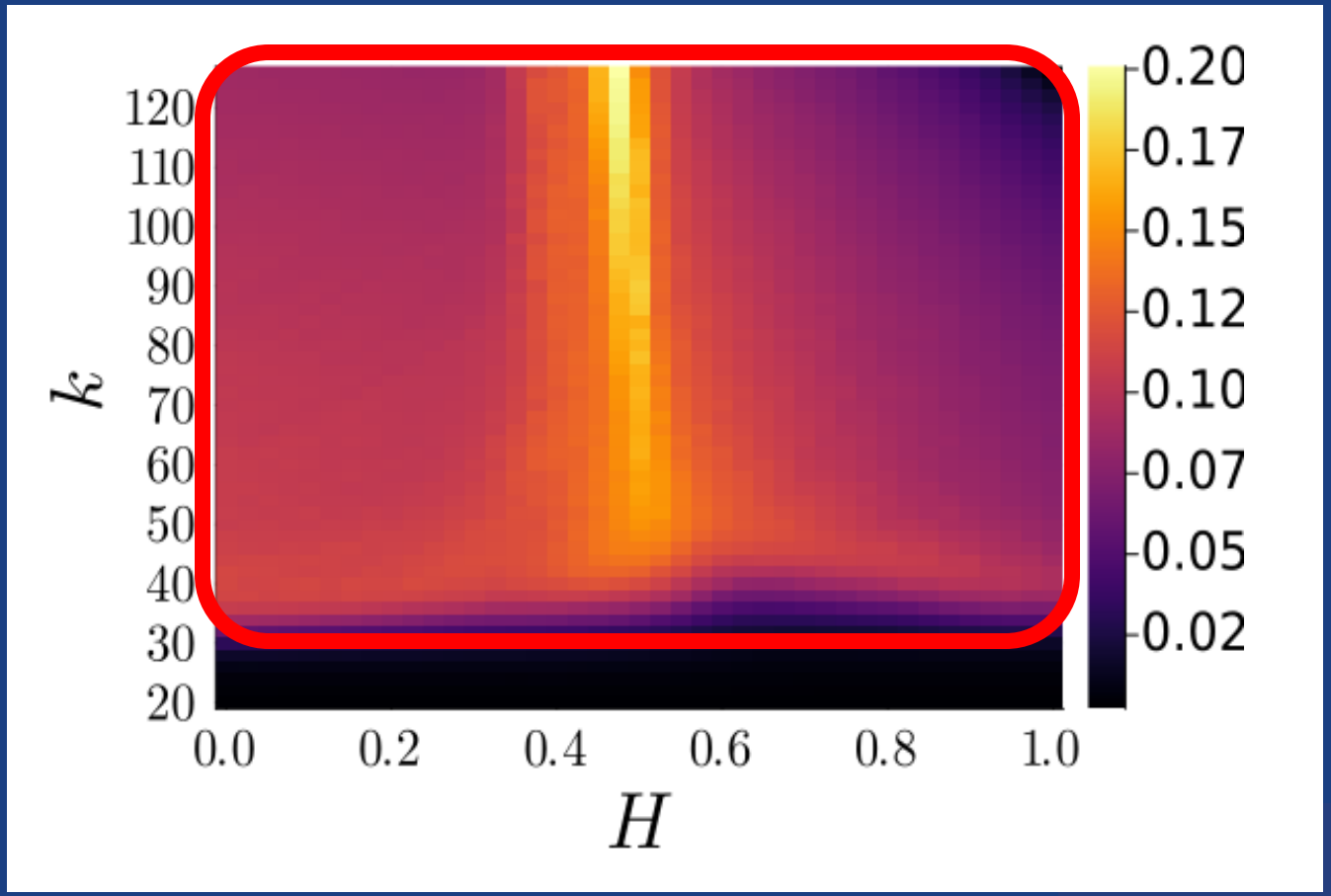
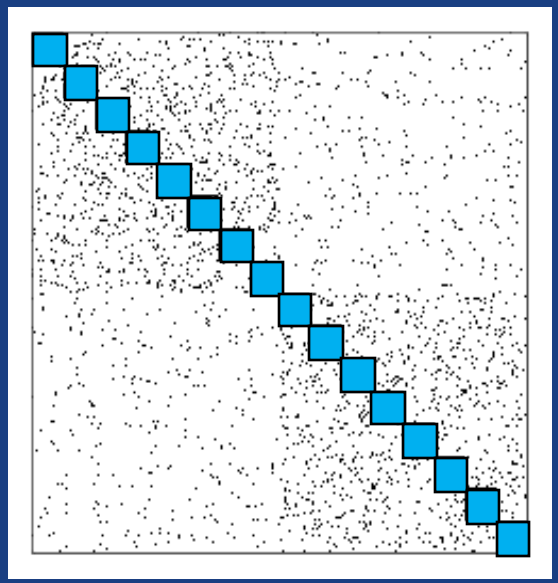


METASTABLE DYNAMICS EMERGES AT THE MODULES' LAYER





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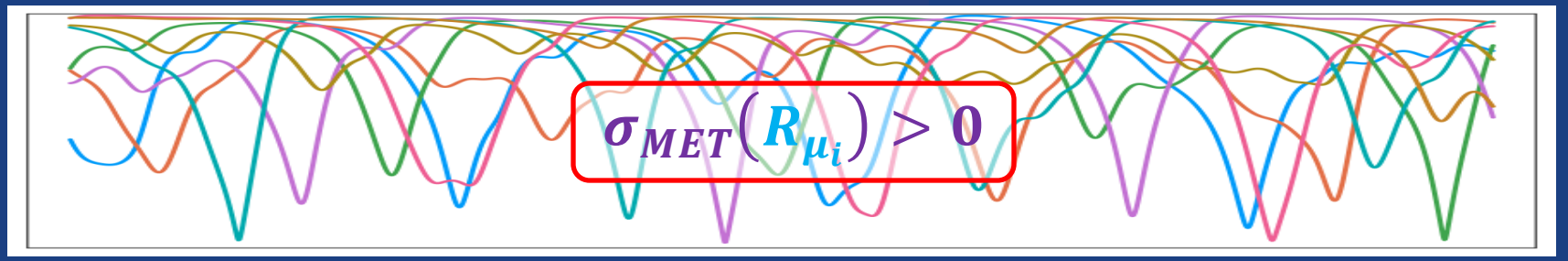


NETWORK PARAMETERS AFFECT THE SPECTRAL GAPS

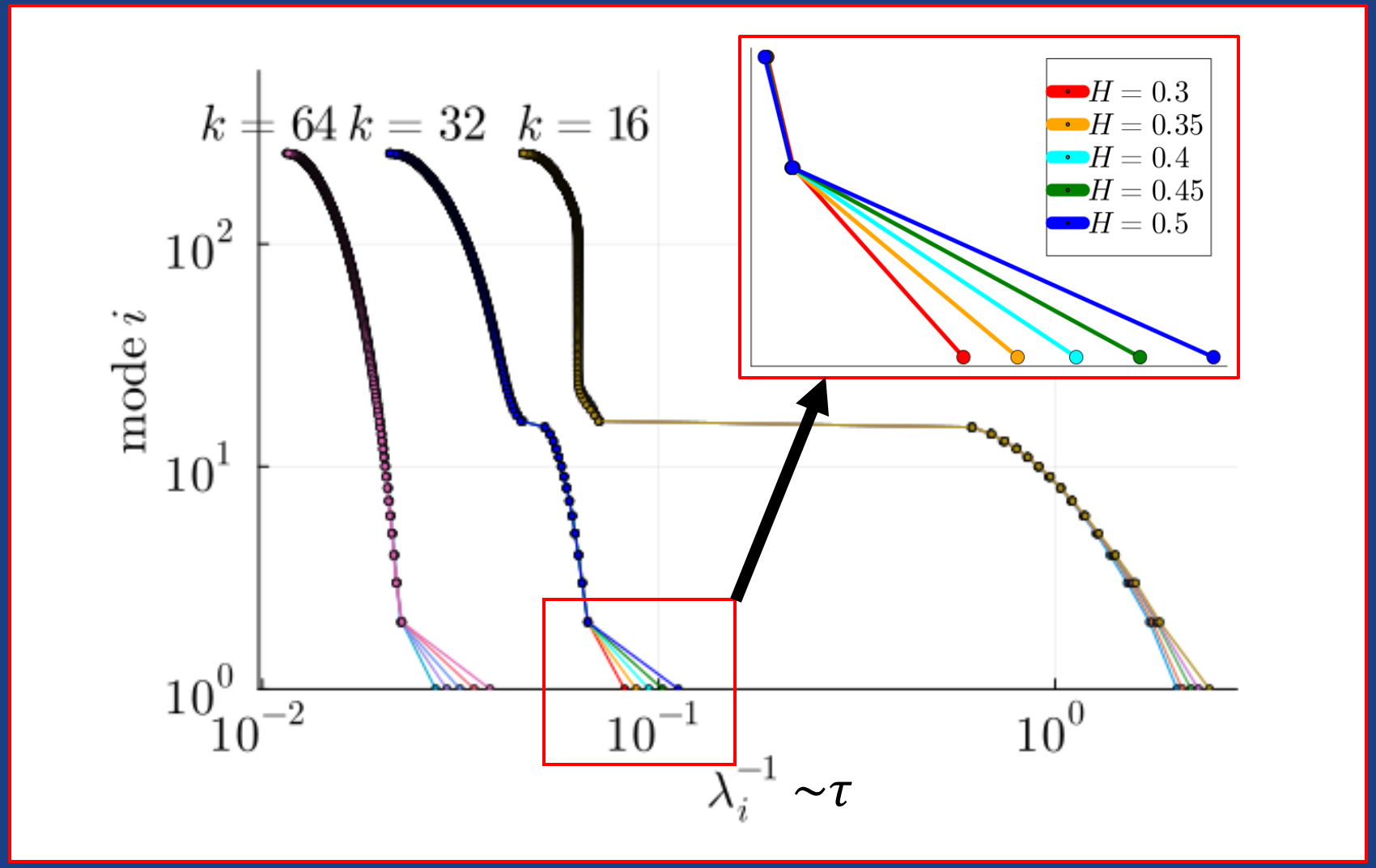
Connectome-Harmonic decomposition framework – Atasoy and colleagues

Laplacian Renormalization Group - Villegas and colleagues

NETWORK PARAMETERS AFFECT THE SPECTRAL GAPS

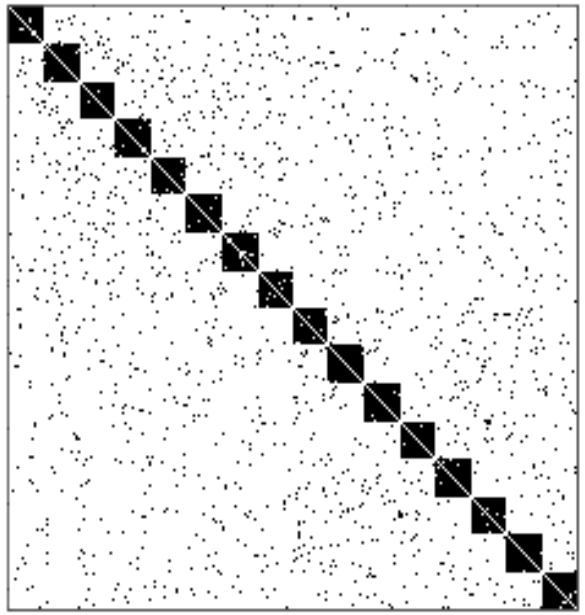
H is related to 1st spectral gap (population layer)

k is related to the 2nd spectral gap (modules layer)



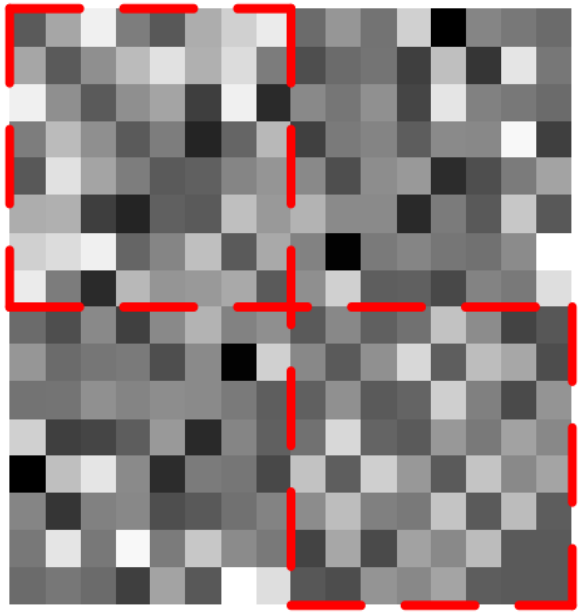
SLOW MODES DETERMINE THE MACROSCOPIC DYNAMICS

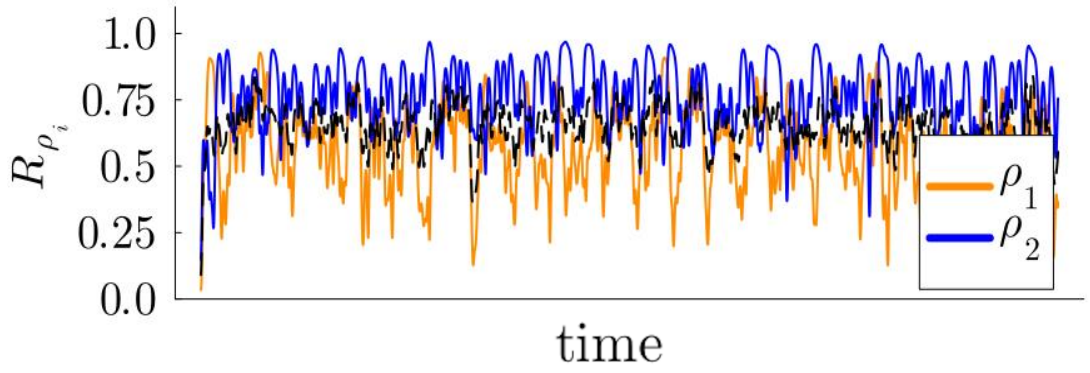
(iff there are two peaks in specific heat)



$$L' = \sum_{i<B_1+1} \lambda_i |\lambda_i\rangle\langle\lambda_i|$$

$$A'_{\alpha\beta} = -\langle\alpha|L'|\beta\rangle$$





$$a' = \frac{K_1 - K_2}{K_1 + K_2} \approx 0.23$$

SLOW MODES DETERMINE THE MACROSCOPIC DYNAMICS

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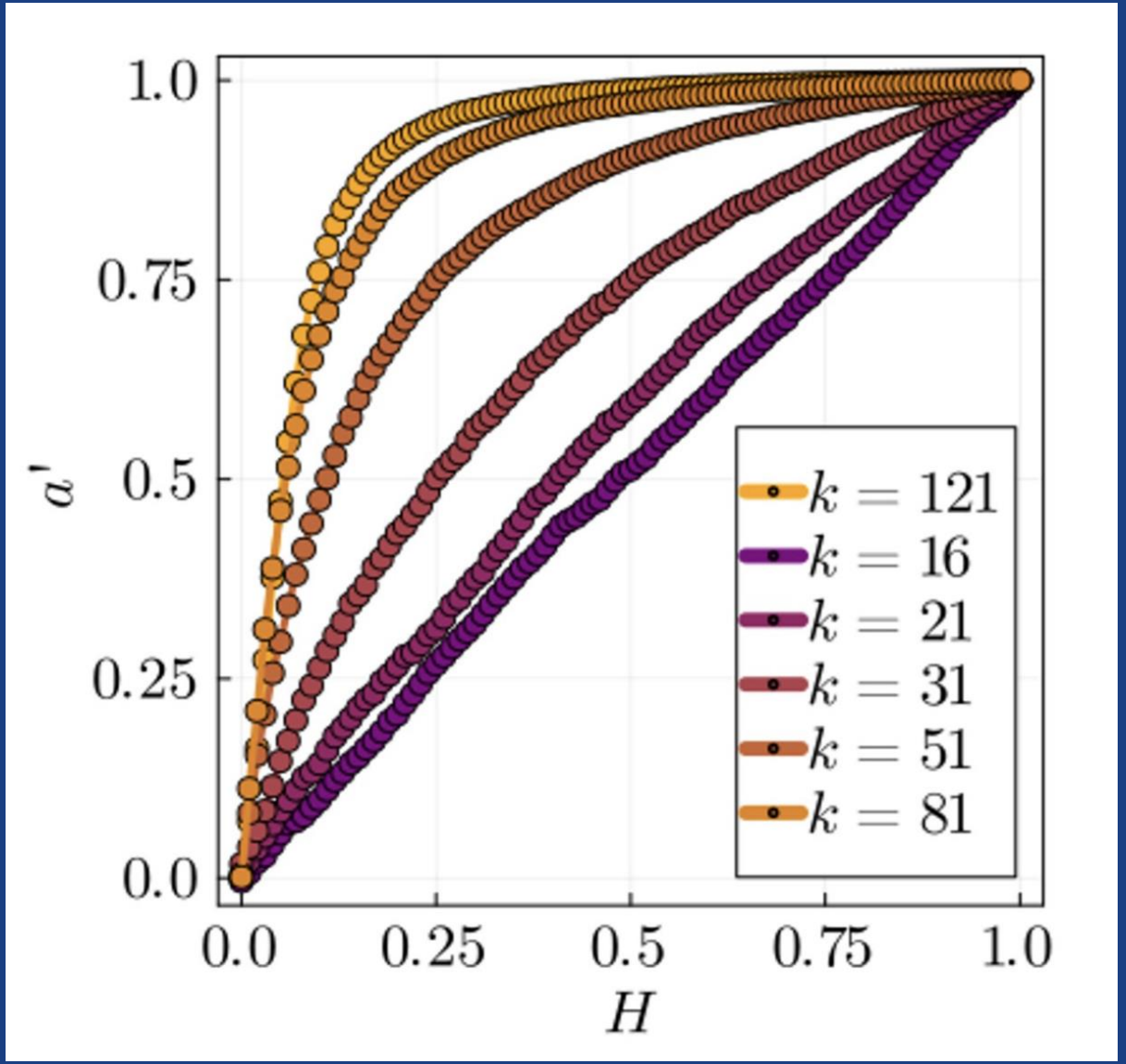
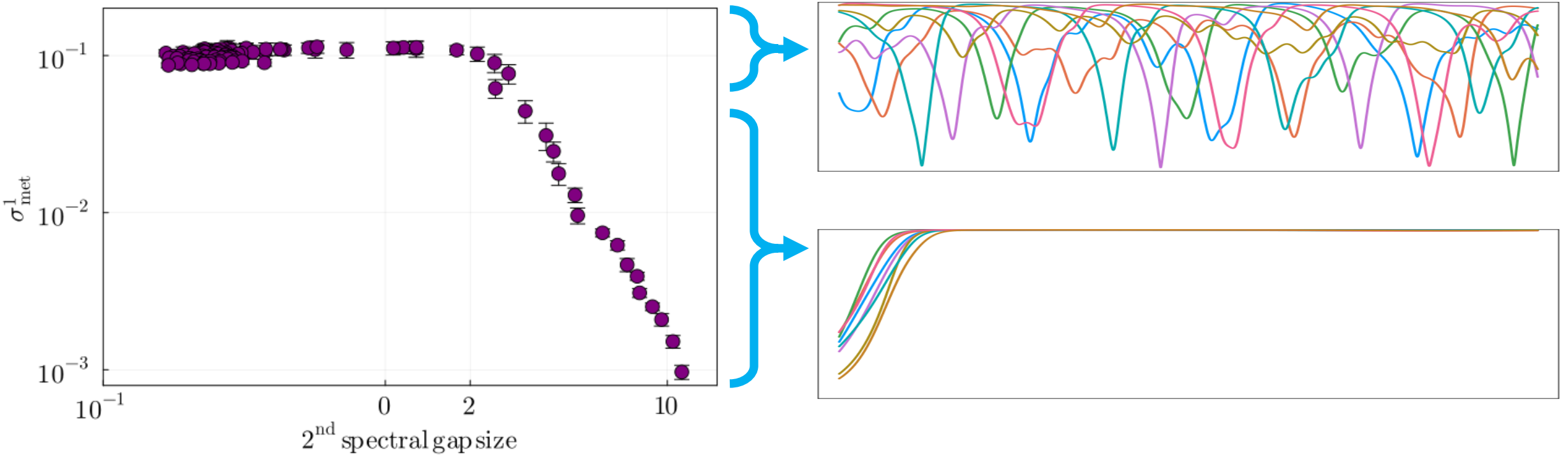


“ENSLAVED” [4] FAST MODES FLUCTUATE DEPENDING ON THE 2ND SPECTRAL GAP (i.e., distance between peaks in specific heat)



[4] Haken, H. (1983). Advanced Synergetics. In Springer Series in Synergetics. Springer Berlin Heidelberg.
[5] Mackay, M., Huo, S., & Kaiser, M. (2023). Spatial organisation of the mesoscale connectome: A feature influencing synchrony and metastability of network dynamics. In B. S. Gutkin (Ed.), PLOS Computational Biology

SO BASICALLY…

We found two distinct pathways to achieve metastability

- Instability of chimera states (similar to the OG Shanahan model)
- Fluctuations of the “enslaved” local order parameters

There exist an explicit relationship between the eigenmodes of the system and metastable states

The Laplacian renormalization group is quite good at identifying the relevant local order parameters

ANALYTICAL RESULTS?

Ott-Antonsen reduction approach is not obvious!

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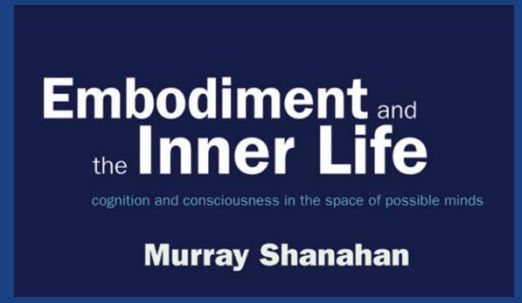
Ott-Antonsen reduction approach is not obvious!



FURTHER WORK?

Software in the natural world: A computational approach to hierarchical emergence

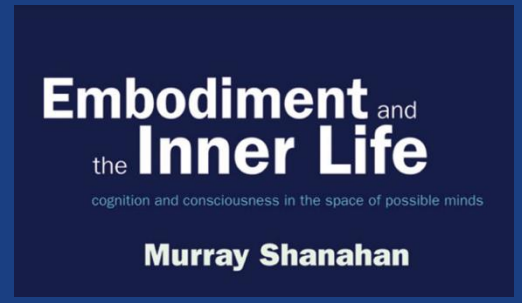
Fernando E. Rosas, Bernhard C. Geiger, Andrea I Luppi, Anil K. Seth, Daniel Polani, Michael Gastpar, Pedro A.M. Mediano

Is metastability an actual emergent process?

→ Solve the information bottleneck problem in which the Lagrange multipliers depend on the eigenvalues of the density matrix

ANALYTICAL RESULTS?

Ott-Antonsen reduction approach is not obvious!



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Software in the natural world: A computational approach to hierarchical emergence

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Is metastability an actual emergent process?

→ Solve the information bottleneck problem in which the Lagrange multipliers depend on the eigenvalues of the density matrix

Address limitations: do these observations hold with the addition of structural heterogeneities (rich club, core-periphery etc)

THANK YOU!

All code and additional information available at:
https://enricocaprioglio.github.io/Lucciole/projects/

No need to know how to code! Just download the .html file and open it in your favourite browser to play with the model.

Otherwise: https://github.com/EnricoCaprioglio/Emergence-of-metastability-in-frustrated-oscillatory-networks

You will also find:

- Density matrix formalism
- Coalition entropy
- Modulation of metastable states
- Heterogeneous frequencies study
- Structural perturbations study
- Derivation of the variation of the nested stochastic block model

moogsoft
Continuous Assurance

UNIVERSITY OF SUSSEX

Luc Berthouze

Ryan Singh working on the Information bottleneck

Simon Farmer, UCL

@Lucciole@mathstodon.xyz
x.com/Enri_Capri @enricocaprioglio@bsky.social

Preprint: https://arxiv.org/abs/2405.14542

TAHNK YOU VERY MUCH!

Whole-system level analysis